III. *A Table of the Equations to equal Altitudes, for the Latitude of the University of* Cambridge, 42° 23′ 28″ N. *with an Account of it's Construction and Use. By the Reverend* JOSEPH WILLARD, *President of the University.*

THE regulation of a clock, which is made ufe of for aftronomical purpofes, is of the utmoft importance. Unlefs it's going is accurately afcertained, the obfervations made by it, however excellent in other refpects, can be of no ufe. Every thing, therefore, which tends to facilitate it's regulation, is worthy of attention.

The beft method to afcertain the going of a clock, where a perfon has not a tranfit inftrument fixed in the meridian, is by equal altitudes of the fun, taken by fome inftrument adapted to the purpofe. Hadley's octant is the moft eafily obtained; and double altitudes may be taken by it, by reflection from a bowl of fome liquid, which will not be eafily put into motion by the air. The oil of tar, or very clean molaffes will anfwer the purpofe well.

The method is, to take the altitude of the fun's upper or lower limb, or both, in the morning as far from noon as may be convenient, and note the time by the clock to a fecond. The time muft be noted in the afternoon, when the altitude is the fame. Then, add half the interval between the two obfervations, to the time of the morning obfervation, which will give the time by the clock, nearly, when the fun's center paffed the

the meridian. This would be the apparent noon exactly, if the ſun did not alter his declination : But as this is conſtantly varying, a ſmall equation, ariſing from the change of declination between the forenoon and afternoon obſervations, muſt be applied to the time of noon thus found, except at the ſolſtices, when the variation is too ſmall to make any equation neceſſary.

ILLUSTRATION.

In Plate I. Fig. VI. let EPQL be the hour circle of ſix o'clock, and ECQ a portion of the Equator. Let P be the north-pole, L the ſouth ; and PZL the meridian of ſome place. Let the arc *lZt* mark the latitude of a given place : Then, the point Z will be it's zenith ; and the arc HOR will be a portion of the horizon, O being 90° diſtant from Z. Let the arc *mxg* mark the ſun's declination, in the morning, at a time when his altitude is taken, and *avn* the declination in the afternoon, when his altitude is the ſame as in the morning obſervation. Let the angle ZP☉ be the diſtance of the ſun from the meridian, at the time of the morning obſervation = the half interval between the forenoon and afternoon obſervations, nearly ; then ſide PZ will be the co-latitude of the place, the ſide P☉ the ſun's co-declination, and the ſide Z☉ the ſun's co-altitude or zenith diſtance. Let the co-altitude Z☉ be ſet off from Z to *r*, a point in the arc of the ſun's declination, at the time of the forenoon obſervation, and through *r* draw the arc P*r*L. Let Z☉ alſo be ſet off from Z to S, a point in the arc of the ſun's declination, at the time of the afternoon obſervation, and through S draw the arc PSL. Then, it will be evident,

dent, that when the ſun has the ſame zenith diſtance in the afternoon obſervation that he had in the forenoon, he will be further diſtant from the meridian by the ſpace *pq*, meaſured upon the Equator, equal to the angle *p*P*q*. Biſect the angle ☉PS, and draw the pricked arc P*b*L, (the ſame as adding the half interval to the time of the forenoon obſervation) which ſhews the mean noon; but it is as much after the apparent noon, or the true time of the ſun's paſſing the meridian, as the ſpace C*b* upon the Equator, which is equal to half *pq*. The time anſwering to C*b*, therefore, muſt, in this caſe, be ſubtracted from the mean noon, which will reduce it to the meridian PCL, or, which is the ſame thing, give the true time when the ſun paſſed the meridian, by the clock. This equation, C*b*, may eaſily be found trigonometrically, by the triangles P☉Z and PSZ.

EXAMPLE.

At Cambridge, latitude 42° 23′ 28″ N. ſuppoſe the altitude of the ſun taken April 2, 1783, at 8h. 40′, A. M. i. e. at 3h. 20′, or 50°, from the meridian, and the correſponding altitude taken in the afternoon; required the equation to the correſponding altitudes?

April 2, 1783, at 8h. 40′, A.M. ☉'s declination,	5° 1′ 00″	Com.=P☉	84° 59′ 0″
At 3h. 20′, P. M. ditto,	5 7 22	Com.=PS	84 52 38
Co-latitude,		=PZ	47 36 32
Angle ZP☉ = 3h. 20′		=	50 0 0

In

In the triangle ZP☉, given ſides PZ and P☉, and the includ-ed angle ZP☉, to find ſide Z☉ the co-altitude or zenith diſ-tance of the ſun at the time of obſervation? Zi is perpendicu-lar to P☉.

Radius,		
: Sine co-angle ZP☉	40° 0′ 0″	9 8080675
:: Tangent PZ	47 36 32	10 0396048
: Tangent ſegment Pi	35 9 6	9 8476723
P☉	84 59 0	
Diff. = ſegment ☉i	49 49 54	
Sine Co-Pi	54 50 54	9 9125573
: Sine Co-☉i	40 10 6	9 8095836
:: Sine Co-PZ	42 23 28	9 8287809
: Sine Co-Z☉	32 7 55	9 7258072
Z☉ = ZS	57 52 5	

In the triangle ZPS are given the three ſides to find angle ZPS.

PZ	47 36 32
PS	84 52 38
ZS = Z☉	57 52 5
Sum of ſides	190 21 15
½ ſum	95 10 37, 5
½ ſum—PZ	47 34 5, 5
½ ſum—PS	10 17 59, 5

PZ	Sine 47° 36′ 42″	9	8683857
x PS	Sine 84 52 38	9	9982618
		19	8666475
: Radius square,		20	
:: ½ sum of sides—PZ = Sine	47 34 5, 5	9	8681039
x ½ sum of sides—PS = Sine	10 17 59, 5	9	2523671
: Sine of square of ½ ∠ ZPS	2	19	2538235
Sine of ½ ∠ ZPS	25 3 33, 7	9	6769117
	x 2		
Angle ZPS	50 7 7, 4		
Subtract ZP☉	50		
Remains space *pq*, or angle *p*P*q*	7 7, 4		
	x 4		
Reduced to time,	28″ 29‴, 6		
The ½ of which is C*b* = the equation to the equal altitudes,	14 14, 8		

This method is accurate, but tedious; and to shorten the work, formulas have been invented. One deduced by Mr. William Wales from art. 256 of Simpson's Fluxions is, perhaps, as easy and concise as any. It is divided into two parts. The first is composed of the change in the sun's declination, during half the interval between the observations, multiplied by the co-secant of the sun's horary angle at the times of the observations,

tions, multiplied again by the tangent of the geographical latitude. The ſecond part conſiſts of the ſaid change in the declination, multiplied by the tangent of the ſun's declination, multiplied again by the co-tangent of the ſun's horary angle. When the ſun's declination is north, this ſecond part of the equation is to be ſubtracted from the firſt; but when it is ſouth, it is to be added to it; and the difference or ſum will be the equation to the equal altitudes: Which equation is additive, when the ſun is declining ſouthward; i. e. when the declination is north decreaſing or ſouth increaſing; but when the contrary, it is ſubtractive.

Theſe rules are to be obſerved, when the latitude is north: They muſt be inverted, when it is ſouth.

The exactneſs of this formula will appear, by making uſe of the foregoing example.

Whole change in declination $6' \, 22''$. Half do. $3' \, 11''$ added to $5^\circ \, 1' \, 0''$, the declination at the time of the morning obſervation, gives $5^\circ \, 4' \, 11''$ for ☉'s declination at noon.

Half change of declin.	$3' \, 11'' = 191''$	2 2810334
x Secant of co-hour angle,	$40^\circ \, 0' \, 0$	10 1157460
x Tangent of the latitude,	42 23 28	9 9603952
First part of the equation,	227, 6	2 3571746

Half change declination	191''	2 2810334
x Tangent declination,	5° 4' 11	8 9479970
x Tangent co-hour angle,	40 0 0	9 9238135
Second part of the equation,	14, 2	1 1528439
Which ſubtracted from the firſt part,	227, 6	
Leaves the equat. to the equal altitudes,	213, 4	
	= 3 33, 4	
	x 4	
The equation in time,	14'' 13''', 6	

By this formula was the following table calculated ; but it is to be obſerved, that the ſun's longitude, inſtead of the declination anſwering to it, is put down for the argument, as being the moſt convenient.

A TABLE of the EQUATIONS to equal Altitudes. For Lat. 42° 23′ 28″ N.

☉'s longitude.	Half Interval between the Observations.															
	H. M. II 0	H. M. II 10	H. M. II 20	H. M. II 30	H. M. II 40	H. M. II 50	H. M. III 0	H. M. III 10	H. M. III 20	H. M. III 30	H. M. III 40	H. M. III 50	H. M. IV 0	H. M. IV 10	H. M. IV 20	H. M. IV 30
S. D.	″ ‴	″ ‴	″ ‴	″ ‴	″ ‴	″ ‴	″ ‴	″ ‴	″ ‴	″ ‴	″ ‴	″ ‴	″ ‴	″ ‴	″ ‴	″ ‴
0 — 0	14.25	14.33	14.41	14.49	14.58	15. 7	15.16	15.27	15.40	15.54	16. 7	16.22	16.38	16.55	17.13	17.33
5	13.52	14. 0	14. 9	14.18	14.27	14.37	14.47	14.59	15.12	15.26	15.40	15.56	16.13	16.31	16.51	17.12
10	13.13	13.22	13.31	13.40	13.50	14. 0	14.11	14.24	14.39	14.54	15. 9	15.25	15.43	16. 2	16.23	16.44
15	12.31	12.41	12.51	13. 1	13.11	13.21	13.32	13.46	14. 1	14.17	14.32	14.49	15. 7	15.26	15.47	16.10
20	11.45	11.55	12. 5	12.15	12.26	12.37	12.49	13. 3	13.18	13.33	13.50	14. 7	14.26	14.46	15. 8	15.30
25	10.56	11. 6	11.16	11.27	11.38	11.49	12. 2	12.16	12.31	12.46	13. 3	13.21	13.40	14. 0	14.23	14.46
I — 0	10. 6	10.16	10.26	10.36	10.47	10.59	11.12	11.25	11.40	11.55	12.12	12.30	12.50	13.11	13.33	14.56
5	9. 4	9 23	9.34	9.44	9.55	10. 7	10.19	10.32	10.48	11. 3	11.20	11.38	11.58	12.19	12.40	13. 2
10	8.21	8.30	8.40	8.50	9. 1	9.12	9.25	9.39	9.54	10. 8	10.25	10.42	11. 1	11.21	11.41	12. 4
15	7.27	7.36	7.46	7.55	8. 6	8.17	8.29	8.42	8.56	9.10	9.26	9.44	10. 3	10.22	10.42	11. 3
20	6.35	6.43	6.52	7. 1	7.11	7.21	7.33	7.45	7.59	8.13	8.28	8.44	9. 2	9.20	9.39	9.58
25	5.43	5.50	5.58	6. 7	6.16	6.26	6.37	6.49	7. 1	7.13	7.27	7.41	7.57	8.14	8.32	8.50
II — 0	4.52	4.58	5. 5	5.12	5.20	5.29	5.39	5.50	6. 2	6.13	6.25	6.38	6.52	7. 7	7.23	7.40
5	4. 0	4. 6	4.13	4.20	4.27	4.35	4.43	4.52	5. 2	5.11	5.21	5.32	5.44	5.57	6.11	6.26
10	3.10	3.15	3.21	3.27	3.33	3.39	3.46	3.53	4. 0	4. 7	4.16	4.26	4.36	4.47	4.49	5.12
15	2.22	2.25	2.29	2.33	2.38	2.43	2.48	2.53	2.59	3. 5	3.12	3.19	3.27	3.36	3.46	3.55
20	1.33	1.35	1.38	1.41	1.44	1.48	1.52	1.56	2. 0	2. 3	2. 7	2.12	2.18	2.24	2.30	2.37
25	0.45	0.46	0.47	0.49	0.51	0.53	0.55	0.57	0.59	1. 1	1. 3	1. 5	1. 8	1.11	1.15	1.19
III + 0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0
5	0.45	0.46	0.47	0.49	0.51	0.53	0.55	0.57	0.59	1. 1	1. 3	1. 5	1. 8	1.11	1.15	1.19
10	1.33	1.35	1.38	1.41	1.44	1.48	1.52	1.56	2. 0	2. 3	2. 7	2.12	2.18	2.24	2.30	2.37
15	2.22	2.25	2.29	2.33	2.37	2.42	2.47	2.52	2.59	3. 5	3.12	3.19	3.27	3.36	3.45	3.55
20	3. 9	3.14	3.19	3.24	3.30	3.37	3.44	3.51	3.59	4. 6	4.15	4.26	4.36	4.47	4.49	5.11
25	3.58	4. 4	4.11	4.18	4.25	4.33	4.41	4.49	4.58	5. 7	5.18	5.30	5.42	5.55	6. 9	6.24
IV + 0	4.50	4.57	5. 4	5.11	5.19	5.28	5.38	5.48	5.59	6.10	6.22	6.36	6.50	7. 6	7.20	7.36
5	5.43	5.50	5.58	6. 6	6.15	6.24	6.35	6.46	6.59	7.11	7.25	7.40	7.56	8.13	8.31	8.49
10	6.34	6.42	6.50	6.59	7. 9	7.20	7.31	7.43	7.56	8. 9	8.24	8.40	8.58	9.16	9.35	9.56
15	7.26	7.34	7.43	7.52	8. 2	8.13	8.25	8.38	8.52	9. 6	9.22	9.39	9.57	10.16	10.37	10.59
20	8.19	8.27	8.36	8.46	8.57	9. 9	9.22	9.36	9.50	10. 4	10.21	10.38	10.56	11.16	11.38	11.59
25	9. 9	9.18	9.28	9.39	9.50	10. 1	10.13	10.27	10.42	10.57	11.14	11.31	11.50	12.11	12.33	12.56
V + 0	10. 1	10.10	10.20	10.30	10.41	10.53	11. 6	11.20	11.35	11.49	12. 6	12.24	12.43	13. 3	13.25	13.48
5	10.51	11. 0	11. 9	11.19	11.30	11.42	11.55	12. 9	12.24	12.39	13.56	13.14	13.33	13.53	14.15	14.38
10	11.39	11.48	11.57	12. 7	12.18	12.29	12.41	12.55	13.10	13.24	12.56	13.59	14.18	14.38	15. 0	15.22
15	12.24	12.33	12.42	12.52	13. 3	13.14	13.25	13.38	13.52	14. 8	14.23	14.40	14.58	15.17	15.38	15.59
20	13. 8	13.15	13.23	13.32	13.42	13.53	14. 4	14.16	14.30	14.45	14.59	15.15	15.32	15.51	16.12	16.33
25	13.43	13.51	14. 0	14.10	14.20	14.30	14.40	14.51	15. 4	15.18	15.31	15.46	16. 2	16.20	16.40	17. 1
VI + 0	14.17	14.24	14.32	14.40	14.49	14.59	15. 8	15.19	15.31	15.44	15.57	16.12	16.28	16.45	17. 3	17.22

A TABLE of the EQUATIONS to equal Altitudes. For Lat. 42° 23′ 28″ N.

☉'s longitude.	H. M. II 0	H. M. II 10	H. M. II 20	H. M. II 30	H. M. II 40	H. M. II 50	H. M. III 0	H. M. III 10	H. M. III 20	H. M. III 30	H. M. III 40	H. M. III 50	H. M. IV 0	H. M. IV 10	H. M. IV 20	H. M. IV 30
	Half Interval between the Observations.															
S. D.	″ ‴	″ ‴	″ ‴	″ ‴	″ ‴	″ ‴	″ ‴	″ ‴	″ ‴	″ ‴	″ ‴	″ ‴	″ ‴	″ ‴	″ ‴	″ ‴
VI + 0	14.17	14.24	14.32	14.40	14.49	14.59	15. 8	15.19	15.31	15.44	15.57	16.12	16.28	16.45	17. 3	17.22
5	14.44	14.51	14.58	15. 6	15.15	15.24	15.33	15.43	15.54	16. 6	16.18	16.32	16.47	17. 3	17.20	17.37
10	15. 6	15.12	15.18	15.25	15.33	15.42	15.50	16. 0	16.10	16.21	16.32	16.45	16.59	17.13	17.28	17.43
15	15.21	15.27	15.33	15.39	15.45	15.52	15.59	16. 8	16.18	16.29	16.39	16.51	17. 4	17.17	17.31	
20	15.29	15.33	15.38	15.44	15.50	15.56	16. 2	16.11	16.20	16.30	16.39	16.49	17. 0	17.12	17.25	
25	15.28	15.32	15.37	15.42	15.47	15.53	15.59	16. 5	16.12	16.20	16.28	16.37	16.47	16.57		
VII + 0	15.17	15.21	15.25	15.29	15.36	15.39	15.43	15.49	15.56	16. 3	16.10	16.18	16.26	16.36		
5	14.58	15. 0	15. 3	15. 6	15.10	15.15	15.19	15.24	15.30	15.36	15.41	15.48	15.56			
10	14.30	14.32	14.34	14.36	14.39	14.42	14.45	14.50	14.55	15. 0	15. 4	15.10	15.16			
15	13.46	13.47	13.48	13.50	13.52	13.55	13.58	14. 2	14. 7	14.12	14.16	14.21				
20	12.52	12.53	12.55	12.57	12.59	13. 2	13. 4	13. 7	13.10	13.13	13.15					
25	11.47	11.48	11.49	11.50	11.51	11.53	11.56	11.59	12. 2	12. 5						
VIII + 0	10.31	10.31	10.32	10.33	10.34	10.36	10.38	10.40	10.43							
5	9. 4	9. 4	9. 5	9. 6	9. 7	9. 8	9. 9	9.10	9.12							
10	7.29	7.29	7.30	7.30	7.31	7.31	7.32	7.32	7.33							
15	5.45	5.45	5.45	5.46	5.46	5.46	5.46	5.47	5.47							
20	3.51	3.51	3.51	3.52	3.52	3.52	3.52	3.53	3.53							
25	1.59	1.59	1.59	1.59	1.59	1.59	1.59	1.59	2. 0							
IX — 0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0							
5	1.59	1.59	1.59	1.59	1.59	1.59	1.59	1.59	2. 0							
10	3.51	3.51	3.51	3.52	3.52	3.52	3.52	3.53	3.53							
15	5.45	5.45	5.45	5.46	5.46	5.46	5.46	5.47	5.47							
20	7.30	7.30	7.31	7.31	7.32	7.32	7.33	7.33	7.34							
25	9. 6	9. 6	9. 7	9. 8	9. 9	9.10	9.11	9.12	9.14							
X — 0	10.33	10.35	10.35	10.36	10.37	10.39	10.41	10.43	10.45							
5	11.51	11.52	11.53	11.54	11.56	11.58	12. 0	12. 3	12. 6	12. 9						
10	12.58	12.59	13. 0	13. 1	13. 2	13. 4	13. 5	13. 8	13.12	13.17	13.21					
15	13.53	13.54	13.55	13.56	13.58	14. 0	14. 2	14. 6	14.11	14.16	14.20	14.26				
20	14.34	14.36	14.38	14.41	14.45	14.49	14.52	14.56	15. 0	15. 5	15. 9	15.15	15.22			
25	15. 5	15. 7	15.10	15.13	15.17	15.22	15.26	15.31	15.37	15.43	15.49	15.56	16. 4			
XI — 0	15.25	15.29	15.33	15.37	15.42	15.48	15.53	15.59	16. 6	16.13	16.19	16.27	16.36	16.45		
5	15.35	15.39	15.44	15.49	15.55	16. 1	16. 7	16.15	16.23	16.31	16.35	16.46	16.48	17. 8		
10	15.36	15.41	15.46	15.52	15.59	16. 6	16.13	16.52	16.31	16.41	16.46	16.58	17. 1	17.22	17.34	
15	15.29	15.35	15.41	15.47	15.54	16. 2	16.10	16.19	16.29	16.39	16.49	17. 1	17.14	17.27	17.41	
20	15.14	15.21	15.28	15.36	15.46	15.52	16. 0	16. 9	16.19	16.30	16.41	16.54	17. 8	17.23	17.39	17.56
25	14.53	15. 0	15. 8	15.16	15.25	15.34	15.42	15.52	16. 3	16.15	16.27	16.41	16.56	17.12	17.29	17.48
0 — 0	14.25	14.33	14.41	14.49	14.58	15. 7	15.16	15.27	15.40	15.54	16. 7	16.22	16.38	16.55	17.13	17.33

The uſe of the foregoing table will appear eaſy by an example.

April 12, 1782, the following correſponding double altitudes of the ſun were taken at Cambridge with Hadley's octant.

	D. Alt.	Foren. Obſ.	Afternoon.	Interval.	½ Interval.	Mean Noon.
Sun's upper limb.	54° 00′	7^h 56′ 27″	4^h 11′ 0″	8^h 14′ 33″	4^h 7′ 16½″	12^h 3″ 43″ 30‴
	58 00	8 7 33	3 59 50	7 52 17	3 56 8½	12 3 41 30
Sun's lower limb.	59 20	8 14 18	3 53 0	7 38 42	3 49 21	12 3 39 00
	61 20	8 20 3	3 47 22	7 27 19	3 43 39	12 3 42 30

Mean noon by the clock by the above obſervations,	12 3 41 37
Equation by the table for change of declina. during the ½ interval,	—13 49
Apparent time by clock when ☉'s center paſſed the meridian,	12 3 27 48

Hence, we find that the clock was 3′ 27″ 48‴ too faſt for apparent time.

The mean ½ interval to theſe correſponding altitudes is $3^{h.}$ 54′ 6″. The ſun's longitude at noon, on the above day, was, at Paris, by *Connoiſance des Temps*, $0^{S.}$ 22° 40′; therefore, allowing for the difference of meridians, it was at Cambridge, $0^{S.}$ 22° 52′. Hence, by taking the proportion in the table between the equations for the ½ interval $3^{h.}$ 50′ and $4^{h.}$ 0′, and for the longitudes $0^{S.}$ 20° and $0^{S.}$ 25°, for the ½ interval $3^{h.}$ 54′ 6″ and the longitude $0^{S.}$ 22° 52′, we ſhall find 13″ 49‴, as in this example.

The

The table of equations to equal altitudes is calculated for latitude 42° 23′ 28″; but by adding or ſubtracting the following ſmall equations, or proportional parts of them, the general equations may be found, as far as two degrees in latitude more or two degrees leſs. Theſe ſmall equations are put down with their ſign for the ſun's longitudes, the half intervals and the latitudes to which they are calculated; and they need no explanation.

		½ Intervals. 2h.	4h. 30′		½ Intervals. 2h.	4h. 30′
S. 0 S. 0 S. 0	Lat. 41° 23′ 28″ N	Equ. — 30‴	— 36‴.	Lat. 40° 23′ 28″	Equ. — 59‴	— 1″ 11‴
☉'s long. 0 0 & VI. 0	Lat. 43 23 28	+ 31	+ 37.	Lat. 44 23 28	+ 1 2	+ 1 17
☉'s long. I. 15 IV. 15	Lat. 41 23 28	— 21	— 26.	Lat. 40 23 28	— 42	— 51
VII. 15 & X. 15	Lat. 43 23 28	+ 22	+ 27.	Lat. 44 23 28	+ 45	+ 55